Flaws in the Efficiency Gap

Christopher P. Chambers, Alan D. Miller, and Joel Sobel[♦]

I. Introduction

Gerrymandering is returning to the Supreme Court.[1] For the first time in three decades, a federal court invalidated redistricting legislation on the grounds that it constituted a partisan gerrymander in violation of the Fourteenth Amendment.[2] That court relied, in part, on a new tool—the efficiency gap—which some have touted as the means to "end gerrymandering once and for all."[3] We evaluate this tool and find it wanting. The efficiency gap is neither a cure to the malady of partisan gerrymandering nor even a good idea. Its use by courts may worsen the problem they seek to solve.

The foundation for partisan gerrymandering claims is the 1986 case of *Davis v. Bandemer*,[4] in which a fractured Supreme Court held that partisan gerrymandering is justiciable as a violation of the Equal Protection Clause of the Fourteenth Amendment because it is an attempt to weaken the voting power of a disfavored political party. The ruling has been repeated in every Supreme Court case on partisan gerrymandering since, despite a minority view that partisan gerrymandering should be held to be a political question.[5] However, while these cases have opened the door to challenges, neither *Bandemer* nor any of the subsequent cases rejected any districting laws as constituting an illegal partisan gerrymander.[6]

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[1] Gill v. Whitford, No. 16-1161 (U.S. will argue Oct. 3, 2017).

[2] Whitford v. Gill, 218 F. Supp. 3d 837 (W.D. Wis. 2016).

[3] *Id.* at 903–10. For the quotation, *see* Nicholas Stephanopoulos, *Here's How We Can End Gerrymandering Once and for All*, NEW REPUBLIC (July 2, 2014), https://newrepublic.com/article/118534/. The efficiency gap is introduced in Eric McGhee, *Measuring Partisan Bias in Single-Member District Electoral Systems*, 39 LEGIS. STUD. Q. 55 (2014) and Nicholas O. Stephanopoulos and Eric M. McGhee, *Partisan Gerrymandering and the Efficiency Gap*, 82 U. CHI. L. REV. 831 (2015).

[4] 478 U.S. 109 (1986).

[5] *See* Vieth v. Jubelirer, 541 U.S. 267 (2004) (plurality opinion). The question of justiciability was not revisited in League of United Latin Am. Citizens v. Perry, 548 U.S. 399 (2006).

[6] Partisan gerrymandering should be distinguished from racial gerrymandering, which is an attempt to weaken (or strengthen) the voting power of a racial group. The Supreme Court has rejected districting laws on the grounds that they constitute a racial gerrymander. *See, e.g.*, Cooper v. Harris, 581 U.S. 1455 (2017).

The problem, raised in *Bandemer* and not remedied in subsequent cases, is the lack of a judicially manageable standard for resolving claims of partisan gerrymandering. The Justices recognized that justiciability requires the existence of judicially manageable standards, and that they knew of no such standards, but decided that the mere fact that they knew of no such standard does not mean that one could not exist. And so, the search has continued for a workable standard, the "holy grail of election law jurisprudence."[7]

The latest candidate for this role is the efficiency gap, a mathematical formula developed by Professor Nicholas Stephanopoulos and Dr. Eric McGhee.[8] The efficiency gap attempts to measure partisan gerrymandering on the basis of a concept that they call "wasted votes."[9] Stephanopoulos and McGhee recommend that any districting plan with an efficiency gap above their recommended threshold be held to be presumptively illegal, subject to a second stage of judicial review.[10] The formula is simple and easy to compute: in its simplified form, it can be calculated on the basis of two numbers, the proportions of votes and seats won by a party.[11]

However, while the mathematical formula is simple, it does not appear to be well understood.[12] The efficiency gap does not seem to have been "stress tested" by examining its implications in simple but extreme settings. Our analysis reveals that the efficiency gap contains an implicit form of cost-benefit analysis; when made explicit, the peculiar nature of this form implies that the measure is deeply problematic, and possibly fatally flawed.

Stephanopoulos and McGhee provide guidance on how courts should apply the efficiency gap in practice; however, their advice is highly questionable. Their proposed threshold test for congressional districting plans is particularly troublesome because it treats large states differently from small

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[7] *Whitford*, 218 F. Supp. 3d at 965 (Griesbach, J., dissenting).

[8] The efficiency gap was introduced in McGhee, *supra* note 3, and Stephanopoulos & McGhee, *supra* note 3.

[9] Stephanopoulos & McGhee, *supra* note 3, at 834.

[10] *Id.* at 884–99.

[11] *Id.*

[12] Others study the mathematical properties of the efficiency gap. See, for example, Wendy K. Tam Cho, *Measuring Partisan Fairness: How Well Does the Efficiency Gap Guard Against Sophisticated as Well as Simple-Minded Modes of Partisan Discrimination?*, 166 U. PA. L. REV. ONLINE 17 (2017); Benjamin Plener Cover, *Quantifying Partisan Gerrymandering: An Evaluation of the Efficiency Gap Proposal*, 70 Stan. L. Rev. (forthcoming 2018); Mira Bernstein & Moon Duchin, *A Formula Goes to Court: Partisan Gerrymandering and the Efficiency Gap*, NOTICES OF THE AM. MATHEMATICAL SOC'Y (forthcoming 2017). This article describes problems with the efficiency gap the literature we cite does not emphasize.

states with no apparent justification. All threshold tests used with the efficiency gap, however, suffer from a different problem—there are election results where all district plans would be rejected.

Perhaps the most serious problem with the efficiency gap is that it relies on a simplistic understanding of gerrymandering. It ignores political heterogeneity within political parties and its application can strengthen extremists at the expense of moderates. It can increase political polarization, and can make the weaker party—which the efficiency gap attempts to protect—worse off.

The efficiency gap also ignores the fact that the redistricting process takes place before the outcome of the vote is known. The level of uncertainty in elections is important for at least two reasons. First, as we explain in Part II, the efficiency gap is built on specific assumptions about how partisan redistricting committees choose to redistrict. Economic research casts doubt on whether these assumptions are reasonable in a setting of uncertainty. Second, the efficiency gap does not consider the ex ante knowledge of the districting committee, or even what may be inferred about this knowledge from the election results. Naïve reliance on this measure may lead to noncompetitive elections.

Stephanopoulos and McGhee propose the efficiency gap as the first part of a two-part test. We focus our analysis on the first part of this test, largely because it is closer to our area of expertise. To the extent that the measure is flawed, we do not believe that adding a second part to this test can save it. If a test is over-inclusive (so that it classifies many valid districting plans as presumptively illegal), it fails its primary purpose, to keep courts away from the political thicket of deciding which plans to reject as unconstitutional. If a test is under-inclusive (so that it classifies many gerrymandered plans as legal), partisans will take the test into account while districting in an attempt to avoid the presumption of illegality. Flaws in the design of the efficiency gap may enable district plans that are "safe" from judicial interference but which still suffer from the serious problems that we identify below.

A. What Economics Has to Offer

We are theoretical economists. The efficiency gap is particularly natural for us to study because it relates to two topics that economists have thought about for a long time: measurement[13] and elections.[14] In particular, economists and other social scientists have for a long time attempted to measure aspects of gerrymandering, including district compactness[15] and partisan bias,[16] and have studied the extent to which voting rules are immune from gerrymandering.[17] The critiques of the efficiency gap that we introduce are applications of ideas from these areas of economic theory.

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[13] For general references see JOHN RICHARD HICKS, VALUE AND CAPITAL (1939) (for compensating and equivalent variation); Anthony B. Atkinson, *On the Measurement of Inequality*, 2 J. ECON. THEORY 244 (1970); Gerard Debreu, *The Coefficient of Resource Utilization*, 19 ECONOMETRICA 273 (1951).

[14] *See generally* KENNETH J. ARROW, SOCIAL CHOICE AND INDIVIDUAL VALUES (2d ed. 1963); DUNCAN BLACK, THE THEORY OF COMMITTEES AND ELECTIONS (1958); PETER C. ORDESHOOK, GAME THEORY AND POLITICAL THEORY (1986); Richard D. McKelvey, *Intransitivities in Multidimensional Voting Models and Some Implications for Agenda Control*, 12 J. ECON. THEORY 472 (1976); William H. Riker & Peter C. Ordeshook, *A Theory of the Calculus of Voting*, 62 AM. POL. SCI. REV. 25 (1968).

[15] The compactness literature has focused on the "shapes" of districts, as shape is usually understood to be a symptom of gerrymandering. This approach does not assign a formal meaning to gerrymandering, but rather seeks to minimize a property (compactness and contiguity) commonly associated with it. *See* Christopher P. Chambers & Alan D. Miller, *A Measure of Bizarreness*, 5 Q.J. POL. SCI. 27 (2010) (introducing the path-based measure of compactness); Christopher P. Chambers & Alan D. Miller, *Measuring Legislative Boundaries*, 66 MATH. SOC. SCI. 268 (2013) (extending the path-based measure of compactness to account for traditional districting criteria); Roland G. Fryer, Jr. & Richard Holden, *Measuring the Compactness of Political Districting Plans*, 54 J. LAW & ECON. 493 (2011) (introducing a measure of compactness of entire districting plans instead of isolated districts and taking an axiomatic approach); Clemens Puppe & Attila Tasnádi, *Axiomatic Districting*, 44 SOC. CHOICE & WELFARE 31 (2015) (defining normative principles that a measure of gerrymandering should satisfy and taking an axiomatic approach); Attila Tasnádi, *The Political Districting Problem: A Survey*, 33 SOC. & ECON. 543 (2011) (providing an overview of the computer science and social science literatures); H. Peyton Young, *Measuring the Compactness of Legislative Districts*, 13 LEGIS. STUD. Q. 105 (1988) (describing and criticizing existing compactness measures).

[16] Roughly, the papers on partisan bias set up a statistical model based on observed vote share and seats, estimate the model, and then extrapolate a "seats-votes" curve from the data. The curve maps vote shares of each party into expected number of seats, and asymmetry of this curve is evidence of partisan bias. *See generally* Andrew Gelman & Gary King, *A Unified Method of Evaluating Electoral Systems and Redistricting Plans*, 38 AM. J. POL. SCI. 514 (1994) (developing a new unified statistical method based on district vote shares); Andrew Gelman & Gary King, *Estimating the Electoral Consequences of Legislative Redistricting*, 85 J. AM. STAT. ASS'N 274 (1990) (developing measures of partisan bias); Gary King, *Representation through Legislative Redistricting: A Stochastic Model*, 33 AM. J. POL. SCI. 787 (1989) (developing a stochastic model of voting that predicts observed features of elections). Similar in spirit is the recent work of Samuel S.-H. Wang. *See* Samuel S.-H. Wang, *Three Tests for Practical Evaluation of Partisan Gerrymandering*, 68 STAN. L. REV. 1263 (2016) (proposing three tests for partisan gerrymandering); Samuel S.-H. Wang, *Three Practical Tests for Gerrymandering: Application to Maryland and Wisconsin*, 15 ELECTION L.J. 367 (2016) (applying data to these three tests). These methods are statistical, and involve evaluating hypothetical election results, extrapolating from observed data.

[17] *See* Sebastian Bervoets & Vincent Merlin, *Gerrymander-Proof Representative Democracies*, 41 INT. J. GAME THEORY 473 (2012) (proving a result related to that of Nermuth, *infra*); Sebastian Bervoets & Vincent Merlin, *On Avoiding Vote Swapping*, 46 SOC. CHOICE WELFARE 495 (2016) (showing that no reasonable rules can avoid vote swapping); Sebastian Bervoets, Vincent Merlin, & Gerhard J. Woeginger, *Vote Trading and Subset Sums*, 43 OPERATIONS RES. LETTERS 99 (2015) (showing that vote trading is computationally complex for three or more parties); Mihir Bhattacharya, *Multilevel Multidimensional Consistent Aggregators*, 46 SOC. CHOICE WELFARE 839 (2016) (analyzing similar problems

As our arguments are grounded in normative economics, it makes sense for us to explain the "axiomatic approach," the dominant mode of analysis in this area.[18] The axiomatic approach imposes normatively attractive restrictions on all methods of measurement and then attempts to characterize the measure or measures that satisfy these properties. In this paper, we describe some normatively attractive properties that the efficiency gap does not satisfy. This approach is abstract in the sense that one formulates the attractive properties for a wide range of situations, rather than treating only a small set of examples. The approach permits us to test rules in hypothetical situations that are unlikely to arise, but which indicate clearly undesirable features of the efficiency gap.

For example, one such situation that we use is a scenario in which an entire state consists of Republican voters. In this situation, gerrymandering for partisan gain is clearly impossible; all districts will be won by Republicans, regardless of the districting plan. As gerrymandering is impossible, a reasonable measure of partisan bias would assign a low score, regardless of the districting plan. However, as we demonstrate below, the efficiency gap leads to the opposite conclusion.[19]

We do not claim that a state composed exclusively of Republican voters is likely in practice. The purpose of the example is to strip away confounding information, so as to present us with a bare bones environment. Just as physical experiments are designed to allow scientists to test their theories under controlled conditions, thought experiments allow us to test our intuitions in an environment in which there is an unmistakable answer. For this reason, our criticism does not depend on the existence of fully Republican states. A failure of the efficiency gap in this extreme case indicates that it cannot be trusted to function well in more realistic environments.

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where the technical assumptions differ); Christopher P. Chambers, *Consistent Representative Democracy*, 62 GAMES & ECON. BEHAV. 348 (2008) (provides generally negative results for single-member districting systems); Christopher P. Chambers, *An Axiomatic Theory of Proportional Representation*, 144 J. ECON. THEORY 375 (2009) (establishing that generalized systems of proportional representation may emerge when moving away from single member districts); Manfred Nermuth, *Two-Stage Discrete Aggregation: The Ostrogorski Paradox and Related Phenomena*, 9 SOC. CHOICE WELFARE 99 (1992) (proving an impossibility theorem).

[18] *See generally* HERVÉ MOULIN, AXIOMS OF COOPERATIVE DECISION MAKING (1988). For axiomatic papers in law, *see* Alan D. Miller & Ronen Perry, *A Group's a Group, No Matter How Small: An Economic Analysis of Defamation*, 70 WASH. & LEE L. REV. 2269 (2013); Alan D. Miller, *Community Standards*, 148 J. ECON. THEORY 2696 (2013); Alan D. Miller & Ronen Perry, *Good Faith Performance*, 98 IOWA L. REV. 689 (2013); Alan D. Miller & Ronen Perry, *The Reasonable Person*, 87 N.Y.U. L. REV. 323 (2012); Matthew L. Spitzer, *Multicriteria Choice Processes: An Application of Public Choice Theory to* Bakke, *the FCC, and the Courts*, 88 YALE L.J. 717 (1979).

[19] *See infra* Part II.A.

In Part II, we describe the efficiency gap and the model of "packing and cracking" on which it is founded. In Part III, we assess whether the efficiency gap is a good test of partisan gerrymandering under the assumptions of the packing and cracking model. In Part IV, we analyze whether reliance on the packing and cracking model makes the efficiency gap less applicable in practice. We then offer a conclusion.

II. Packing, Cracking, and the Efficiency Gap

The efficiency gap is a measure of electoral outcomes proposed as a tool to identify partisan gerrymandering. The efficiency gap was relied upon, in part, by the district court in *Whitford v. Gill*, which found it to be evidence of partisan gerrymandering when upholding a challenge to redistricting legislation enacted by the state legislature.[20] In this section, we explain the efficiency gap and the assumptions upon which it relies.

We begin with a simple model of districting that is commonly used to explain gerrymandering. In the model, a state consists of a set of voters; these voters come in two types: Democrats and Republicans. The state must be divided into a certain number of equipopulous legislative districts.[21] A partisan districting committee must decide how to allocate voters to districts.

This is a very simple model. There is no uncertainty; the districting committee knows precisely where each voter is located, and for whom he or she will vote. Every voter goes to the polls on election day and votes exactly as the districting committee predicts. There are no progressives or "Blue Dogs," but only Democrats; there are no neoconservatives or paleoconservatives, but only Republicans. There are no geographical constraints; the districting committee has complete flexibility in allocating voters into districts.

In this model, the partisan districting committee has the objective of maximizing the number of seats that its party obtains. To achieve the objective, it generally employs two techniques. The first such technique, the *concentration gerrymander,* involves "packing" the opponent party's supporters into districts where they form large majorities, far in excess of what is necessary for them to win the districts. The second such technique, called the *dispersal gerrymander,* involves spreading out, or "cracking," the opponent

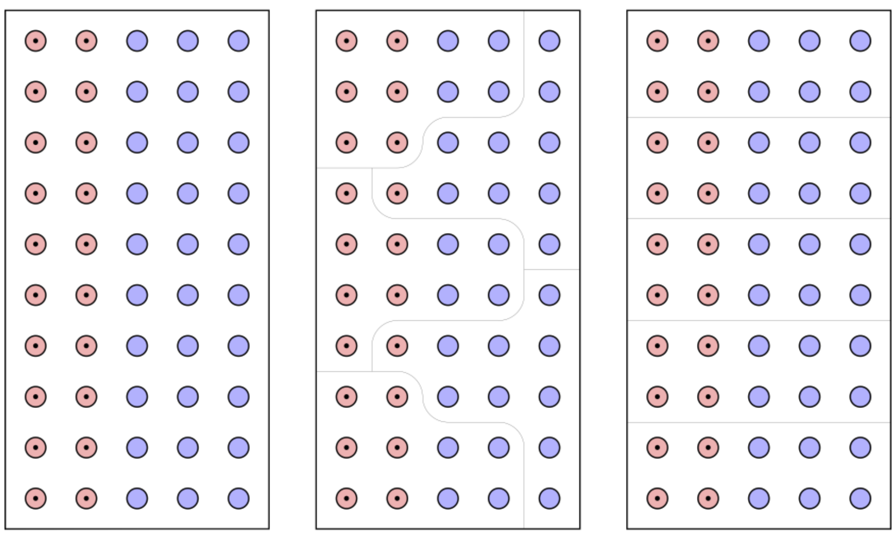
[20] 218 F. Supp. 3d 837 (W.D. Wis. 2016).

[21] The requirement that districts be equipopulous comes from Reynolds v. Sims, 377 U.S. 533, 568 (1964).

party's supporters so that they form the minority of districts where the districting committee's party has a slight majority.[22] For what follows, we will use the term "packing and cracking" to refer to the combined set of assumptions: that partisan gerrymandering is best understood as the use of these techniques, in the context of the model described above, to help the districting committee's preferred party win more seats.[23]

To make packing and cracking easier to understand, we provide a simple example.[24] Figure 1(a) depicts a state with fifty voters, to be divided into five districts of ten voters each. Twenty of the voters (or 40%) are Republicans (represented by red with black dots in the center), while thirty of the voters (60%) are Democrats (represented by blue).[25]

Figure 1



(a) Before Districting (b) Republican Plan (c) Democratic Plan

[22] These techniques are described in Guillermo Owen & Bernard Grofman, *Optimal Partisan Gerrymandering*, 7 POL. GEOGRAPHY Q. 5 (1988).

[23] It is important for us to specify that we are referring to packing and cracking in the context of this model; the techniques can mean something different in the context of other models.

[24] Figure 1 is adapted from Christopher Ingraham, *This Is the Best Explanation of Gerrymandering You Will Ever See*, WASH. POST (Mar. 1, 2015), https://www.washingtonpost.com/news/wonk/wp/2015/03/01/this-is-the-best-explanation-of-gerrymandering-you-will-ever-see/?utm_term=.464bb2e3a7c3.

[25] For simplicity, we assume in these examples that the Democrats are the larger party. Residents of red states can simply switch the partisan labels. For readers who cannot see color, the Republicans are represented by circles with the black dots.

Figure 1(b) describes an optimal plan from the perspective of the Republicans.[26] It contains two heavily Democratic districts, created by packing, and three districts where Republicans have a slight majority, created by cracking. This plan results in the Republicans winning three out of five seats (60%) despite the fact that they have only 40% of the votes.

Figure 1(c) describes an optimal plan from the perspective of the Democrats. They win all five seats even though they have only 60% of the vote. This plan was created through cracking, but not packing.

How can a party achieve an electoral outcome significantly in excess of its vote share? Under the packing and cracking model, the key is to realize that once a party has a majority of the votes in a district, all other votes are irrelevant to the outcome of the election. To use the term of Stephanopoulos and McGhee, these irrelevant votes are "wasted."[27] Packing results in the opponent's party wasting votes because the packed districts contain many more of the opponent's supporters than necessary to win the election. Cracking results in the opponent's wasting votes because the cracked districts contain a large minority of the opponent's supporters who end up on the losing side in the race.

In the view of Stephanopoulos and McGhee, partisan gerrymandering is packing and cracking.[28] According to this logic, a successful gerrymander wastes as few votes of the favored party as is possible. Consequently, the efficiency gap is essentially the number of wasted votes—"a tally of all the cracking and packing decisions in a district plan."[29]

The formal definition of the efficiency gap is slightly more complicated than the number of wasted votes. To calculate it, one must first calculate the numbers of votes wasted by each of the two parties, and then take the differ-

[26] There are several such optimal plans; all result in the Republicans winning three out of the five districts.

[27] Stephanopoulos & McGhee, *supra* note 3, at 834 (defining a vote as "wasted" if "it is cast (1) for a losing candidate, or (2) for a winning candidate but in excess of what she needed to prevail.").

[28] *Id*. at 851–52 (noting that "some kind of cracking and packing is how all partisan gerrymanders are constructed" and that critics of partisan gerrymandering "typically conceive of gerrymandering as the systematic disadvantaging of a party through the cracking and packing of its supporters.").

[29] *Id*. at 852.

ence between these two numbers. The efficiency gap is this difference divided by the total number of votes. [30] For any specific election, the efficiency gap results in the same ranking of plans as the number of wasted votes.[31]

We demonstrate the efficiency gap in Table 1.

For the Republican plan (in Figure 1(b)), there are two types of districts; three where the Republicans have six votes and the Democrats have four, and two where the Republicans have one and the Democrats nine. In the former districts, all six Republican votes are necessary to win the district,[32] so the wasted votes are those of the four Democrats; in the latter districts, only six of nine Democratic votes are necessary to win the district, so three of the Democratic votes and the one Republican vote are counted as wasted. This leads to a total of two wasted Republican votes and eighteen wasted Democratic votes; because there are more wasted Democratic votes, we subtract the former from the latter and we get a net of sixteen wasted Democratic votes. Dividing by the total number of votes cast (fifty) gives us the efficiency gap, which in this case is 32% (in favor of the Republicans).

For the Democratic plan (in Figure 1(c)), we repeat the exercise. Here, for all five districts, the Republicans have four votes and the Democrats have six. The result is that, in each district, it is the four Republican votes that are wasted, as all six Democratic votes are necessary to win the district. This leads to twenty wasted Republican votes. Because there are no wasted Democratic votes, we simply divide this number by the total number of votes cast, resulting in an efficiency gap of 40% (in favor of the Democrats).

[30] This refers to the efficiency gap as defined in McGhee, *supra* note 3, and Stephanopoulos & McGhee, *supra* note 3. However, there exist other versions; *see* Eric McGhee, *Measuring Efficiency in Redistricting*, 16 ELECTION L.J. (forthcoming 2017), http://online.liebertpub.com/doi/pdf/10.1089/elj.2017.0453, which introduces a modified version of the efficiency gap to account for a perceived problem of the original measure. (The problem is that the efficiency gap may fail to satisfy McGhee's "efficiency principle" when districts do not have equal numbers of voters. When districts do have equal numbers of voters, the original and modified measures coincide.)

[31] The two definitions result in the same ranking of plans because the total number of wasted votes is independent of the outcome of the vote. This implies that the number of wasted Republican votes can be determined by knowing the number of wasted Democratic votes; it is simply the total number of wasted votes minus the number of wasted Democratic votes. The extra complications in the efficiency gap formula are there only to make the efficiency gap scores easier to understand and to provide a semblance of comparability across different elections and different states. In mathematical language, we would say that the efficiency gap is "normalized."

[32] We ignore the possibility of a tie. In practice, ties are unpredictable and extremely rare.

Table 1

		Number of Votes		**Wasted Votes**	
		Republicans	Democrats	Republicans	Democrats
Republican Plan	**District 1**	6	4	0	4
	District 2	6	4	0	4
	District 3	6	4	0	4
	District 4	1	9	1	3
	District 5	1	9	1	3
	Total	**20**	**30**	**2**	**18**
	efficiency gap: (18-2) ÷ 50 = 32% in favor of the Republicans				
Democratic Plan	**District 1**	4	6	4	0
	District 2	4	6	4	0
	District 3	4	6	4	0
	District 4	4	6	4	0
	District 5	4	6	4	0
	Total	**20**	**30**	**20**	**0**
	efficiency gap: (20-0) ÷ 50 = 40% in favor of the Democrats				

Now that we have computed the efficiency gap, what is to be done with our results? Stephanopoulos and McGhee recommend that a districting plan be held presumptively invalid if the efficiency gap exceeds an 8% threshold for state legislative plans, or a two-seat threshold for congressional plans.[33] In the case of a five-district state depicted in Figure 1, the two-seat threshold is 40%.[34] Thus, while both of these plans exceed the threshold for state legislative plans, only the Democratic plan reaches the threshold for congressional plans.

[33] Stephanopoulos & McGhee, *supra* note 3, at 837, 884.

[34] To see why, let **PV** be the vote share, and let **PS** = $\mathbf{M}/\mathbf{N}$, where **M** is the number of seats for which the efficiency gap is zero, and where **N** is the number of districts. Then, by the simplified efficiency gap

If all districts contain an equal number of voters, and all voters vote for one of the two parties, the efficiency gap calculation can be greatly simplified.[35] We need only focus on the case of a single party, and on two measured variables, the proportion of votes won by the party (**PV**) and the proportion of seats won by the party (**PS**). The simplified efficiency gap in favor of that party, then, is:[36]

$$\left(\mathbf{PS} - \frac{1}{2}\right) - 2\left(\mathbf{PV} - \frac{1}{2}\right).$$

For simplicity, we work with the assumption that all districts contain equal numbers of voters. This assumption allows us to use the simplified efficiency gap calculation when dealing with large numbers of voters. With one exception, all of our examples apply to the more general case, if appropriately modified. That exception involves the calculation of the two-seat threshold proposed for congressional plans; without the assumptions, it is not clear how to calculate whether a plan exceeds this threshold.

formula below, we have ($\mathbf{M}/\mathbf{N}$ – ½) – 2 (**PV** – ½) = 0. We want to find the efficiency gap when the party wins two extra seats; this is simply ($\mathbf{M}+2/\mathbf{N}$ – ½) – 2 (**PV** – ½). Because ($\mathbf{M}/\mathbf{N}$ – ½) – 2 (**PV** – ½) = 0, this reduces to $2/\mathbf{N}$. For **N** = 5, this is 40%. It is not clear how to calculate the two-seat threshold without this formula.

[35] Stephanopoulos & McGhee, *supra* note 3, at 853. They write that this simplification can be done whenever all districts have equal populations, as is constitutionally required; however, this is incorrect. The required assumption is actually that the number of voters be the same across districts. In practice, this may not be the case despite the constitutional requirement of equipopulous districts. For example, if we look at congressional elections in California, more than two and a half times as many votes were cast in 2016 in the fourth district (350,978) than in the twenty-first (132,408); this despite the fact that the districts had nearly equal populations at the time of the 2010 census. Similarly, more than four times as many votes were cast in 2014 in the second district (217,524) than in the fortieth district (49,379). *See* ALEX PADILLA, CAL. SEC'Y OF ST., STATEMENT OF THE VOTE (2016), http://elections.cdn.sos.ca.gov/sov/2016-general/sov/2016-complete-sov.pdf and DEBRA BOWEN, CAL. SEC'Y OF ST., STATEMENT OF THE VOTE (2014), http://elections.cdn.sos.ca.gov/sov/2014-general/pdf/2014-complete-sov.pdf.

[36] Stephanopoulos & McGhee, *supra* note 3, at 853. To see why we can simplify, let **TV** be the total number of votes cast in the election, and let **N** be the number of districts. Then the total number of votes won by the party is given by **PV** × **TV**, and the total number of seats won is given by **PS** × **N**. Consequently, the number of votes wasted by the party is **PV** × **TV** – **PS** × **N** × **TV** ÷ (2 **N**). Because the proportion of votes and seats won by the opposing party is (1 – **PV**) and (1 – **PS**), respectively, it follows that the number of votes wasted by the opposing party is (**1** – **PV**) × **TV** – (**1**– **PS**) × **N** × **TV** ÷ (2 **N**). The efficiency gap in favor of a party is the number of votes wasted by the opposing party minus the number of votes it wastes, divided by the total number of votes, or [((1 – **PV**) × **TV** – (1 – **PS**) × **N** × **TV** ÷ (2 **N**)) – (**PV** × **TV** – **PS** × **N** × **TV** ÷ (2 **N**))] ÷ **TV**; this simplifies to (1 – 2 **PV**) – (1 – 2 **PS**) ÷ 2, which in turn simplifies to (**PS** – ½) — 2 (**PV** – ½).

Our criticism of the efficiency gap takes two parts. In Part III, we ask whether the efficiency gap is a good way to test for partisan gerrymandering, taking as given the assumptions of the packing and cracking model. In Part IV, we ask whether these assumptions are themselves reasonable given the goal of measuring partisan gerrymandering.

III. THE EFFICIENCY GAP AS A MEASURE OF PACKING AND CRACKING

In this Part, we assume, for the sake of argument, that the packing and cracking story is a good explanation of partisan gerrymandering. We then ask whether, given this assumption, we should use the efficiency gap to test for partisan gerrymandering. Our conclusion is that we should not. The efficiency gap relies on a flawed method of cost-benefit analysis; a correction of this method leads us to a different measure. Furthermore, the proposed thresholds for the application of the efficiency gap do not appear to have been carefully thought out.

A. The Benefit of a Seat

The specific method used to measure wasted votes in the context of the efficiency gap is controversial. The authors count all of a party's votes as wasted if the party loses in that district, and if the party wins in that district, the authors count all of the party's votes in excess of the 50% plus one threshold necessary to win the district.[37] Judge Griesbach's dissent in *Whitford v. Gill*, for example, criticized the wasted vote measure by using a sports analogy, noting that the use of a similar method to calculate "wasted runs" in baseball would be commonly understood to be absurd.[38]

The idea that parties want to minimize wasted votes seems natural. Waste is a cost that comes without any benefit; most people want to minimize waste. A wasted vote is a vote that perhaps could have been used in a different district, to gain the party an extra seat.

However, few people work solely to minimize waste; instead, they seek to balance costs with benefits. If votes are a cost, the benefits are seats. To perform a cost-benefit analysis, we need to be able to compare votes and seats. In this section, we make two claims: first, that the efficiency gap relies on an implicit method of comparison, and second, that given the objective of the efficiency gap, the wrong method of comparison is used.

[37] This is 50% plus one-half if the number of voters is odd.

[38] Whitford v. Gill, 218 F. Supp. 3d 837, 958 (W.D. Wis. 2016). The dissent's criticism relies, in essence, on the assumption that turnout is not fixed.

To understand our argument, it will help to focus on how the measure applies to a single party, in a single district. Votes cast for the Republicans can be wasted in two ways. First, all such votes are wasted if the Republicans fail to win the district. Second, should the Republicans succeed, votes are wasted if they are above the 50% plus one threshold required to win the district.

Economists conduct cost-benefit analysis using the concepts of "marginal benefit" and "marginal cost."[39] In our context, the marginal benefit is the gain the party receives from an additional vote in its favor; the marginal cost is the cost of that extra vote. Typically, marginal benefits and costs are measured in terms of dollars, but that is both difficult and unnecessary; it is sufficient to measure these benefits and costs in terms of votes.

The marginal cost of a vote, measured in terms of votes, is always one vote. This part is simple. What is the marginal benefit of a vote? Under the efficiency gap, the marginal benefit of the first vote is zero: one vote is insufficient to win the district. As there is a cost to this vote, but it results in no benefit, it is deemed to be "wasted." The same is true for the second, third, and fourth votes, and for all votes up to (and including) the 50% threshold.

For the first vote that passes the threshold, however, things are different. That vote results in the party capturing the seat. The efficiency gap declares it not wasted, and furthermore, it resets the measure of waste to zero, so that all previous votes are now declared not to have been wasted. This implies that the marginal benefit from capturing the seat is equal to the sum of the marginal costs from all votes up to and including the deciding vote. Measured in terms of votes, then, the benefit of the seat is equal to the cost of 50% plus one of the votes.

Every additional vote is counted by the efficiency gap as a wasted vote. The marginal benefit from these votes is zero; the seat has already been won, so they do no extra good. The marginal cost of these votes is still equal to one vote. The efficiency gap can thus be understood as a measure of the "relative inefficiency" of districting plans.[40] It weights the costs against the benefits for each party and then takes the difference in an attempt to determine, essentially, which party gets a better deal, and by how much. The efficiency gap is not the only possible relative inefficiency measure. There are

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[39] *See, e.g.*, E.J. MISHAN, ELEMENTS OF COST-BENEFIT ANALYSIS (1972); BEN S. BERNANKE & ROBERT H. FRANK, PRINCIPLES OF ECONOMICS 378 (3d ed. 2005).

[40] For inefficiency in other contexts see Debreu, *supra* note 13, and Christopher P. Chambers & Alan D. Miller, *Inefficiency Measurement*, 6 AM. ECON. J.: MICROECONOMICS 79 (2014). Importantly, once the population of voters is known, the total costs for either party are fixed and immutable.

different ways to value the benefit from capturing the seat; each value implies a different relative inefficiency measure.

Let us return to the thought experiment described in the introduction. Imagine a state that consists entirely of a single party; for example, all voters are Republicans. This state would be ungerrymanderable; every conceivable districting plan would result in 100% of the seats being captured by the Republicans. A natural property of a desirable measure is that such a state should be determined to be ungerrymandered. The efficiency gap, however, would declare this state to be heavily gerrymandered in favor of the Democrats: all wasted votes are Republican votes; the Democrats simply have no votes to waste. This state would receive the worst possible score, almost 50%.

To put this in perspective, the efficiency gap treats this state as equivalent to the case where the Democrats win every district by a single vote. To see why, note that in this case, the Democrats still waste no votes; not because they have no votes, but because they have exactly the number needed to win, so none are wasted. All Republican votes are wasted, however, and all wasted votes are Republican votes. So again, this state is judged to be heavily gerrymandered in favor of the Democrats.

The hypothetical state where all voters are Republican is ungerrymanderable; any reasonable measure of partisan gerrymandering should determine it to be ungerrymandered. A measure of relative inefficiency will consider a districting plan to be ungerrymandered if (and only if) the Democrats' net cost is equal to that of the Republicans.[41] The Republicans, on the other hand, pay the maximum cost (they receive all votes) and receive the maximum benefit (they win all seats). Their net cost must be equal to the net cost of the Democrats, and this latter cost must be zero, because the Democrats neither pay any cost (they receive no votes) nor receive any benefit (they win no seats). An implication is that the benefit from winning all seats must exactly equal the cost of receiving all of the votes. And this implies, in turn, that the benefit of a single seat must be equal to all of the votes cast in that district.[42] Recall that the efficiency gap, instead, equated the benefit of a single seat with approximately half of the votes cast in the district.

The idea that the benefit from winning a seat should be equal to the sum of the votes cast in the district, and not simply half of the votes plus one,

[41] The net cost is the cost minus the benefit.

[42] Technically, this must be true on average; as mentioned earlier, we are keeping the assumption that all districts have equal numbers of voters.

makes intuitive sense. The seat contains all of the political power in the district. If a candidate wins an election with 60% of the vote, we think it is more natural to say that this candidate has gained an advantage from the system (having won all of the power with less than full support), than it is to say that this candidate has suffered a loss.

The measure that results from our recalibration of the benefit of a seat would also lead to a natural result in the hypothetical case where the Democrats win every district by a single vote. In this case, the Democrats would receive the maximum possible net benefit; that is, they would receive the maximum possible benefit (all seats), and pay the minimum cost necessary to get this benefit. The Republicans, by contrast, would pay the maximum possible net cost; that is, they would receive no benefit (they win no seats), but pay the maximum cost possible without winning any seats. Because the Democrats' net benefit is as far as is possible from the Republicans' net cost, the measure would in this case yield the same result as the efficiency gap, assigning the worst possible score, and determining the state to be heavily gerrymandered in favor of the Democrats.

The measure that results from our recalibration also leads to a metric that is completely intuitive. Using the same notation as before for the proportion of votes won by a party (**PV**) and the proportion of seats won by that party (**PS**), the resulting measure (in favor of the party) is simply:[43]

$$\mathbf{PS} - \mathbf{PV}.$$

In other words, our recalibration of their metric leads to the difference between the proportion of seats won and the proportion of votes won; as a consequence, it associates the ideal situation with complete proportionality.[44]

We emphasize here that we do not suggest that the different mathematical equation we derive is appropriate for measuring partisan gerrymanders. In

[43] To see why we can simplify, let **TV** be the total number of votes cast in the election, and let **N** be the number of districts. Then the total number of votes won by the party is given by **PV** × **TV**, and the total number of seats won is given by **PS** × **N**. Consequently, the party's net cost is **PV** × **TV** – **PS** × **TV**. Because the proportion of votes and seats won by the opposing party is (1 – **PV**) and (1 – **PS**), respectively, it follows that the net cost of the opposing party is (**1** – **PV**) × **TV** – (1 – **PS**) × **TV**. The measure that results from our recalibration is the net cost of the opposing party minus the cost of the initial party, divided by the total number of votes, or [((1 – **PV**) × **TV** – (1 – **PS**) × **TV)** – (**PV** × **TV** – **PS** × **TV**)] ÷ **TV**; this simplifies to 2 (**PS** – **PV**). Because the measure gives an identical ranking if transformed by a constant, we can simplify this to **PS** – **PV**.

[44] If viewed as a measure of wasted votes, the measure that results from our recalibration would be a counterexample to McGhee's claim, *supra* note 30, that the efficiency gap is the only measure of wasted votes that satisfies his efficiency principle.

fact, we believe it is probably inappropriate. As we explain below, the implicit benefit of a winning seat is only one of several problems with the efficiency gap. Furthermore, the Supreme Court has in the past rejected the idea that proportionality is required by the Constitution. Were proportionality the ideal, it could easily be ensured without the Court's interference by replacing single-member legislative districts with a system of proportional elections.

The efficiency gap is not proportional in this sense; it does not imply that the proportion of seats won should equal the proportion of votes won. Rather, it is quasi-proportional in the sense that it implies that the proportion of seats should be a function of the proportion of votes: specifically, twice the proportion of votes minus 50%. That is, it awards a "winner's bonus" so that an extra percent in the proportion of votes yields the party an extra two percent in the proportion of seats awarded.[45]

Stephanopoulos and McGhee argue that quasi-proportionality is a positive attribute of the efficiency gap. However, it is hard to see why the efficiency gap should be preferred on these grounds. McGhee points out that the Supreme Court cases rejecting the idea that the Constitution requires proportionality have not rejected their claim that the Constitution requires a form of quasi-proportionality,[46] but this issue has likely not been brought before the Court.

Were the winner's bonus implied by the efficiency gap the ideal, it could also be implemented through a quasi-proportional voting system, in which seats are awarded directly on the basis of the efficiency gap formula. Such a system might be politically infeasible because it may be perceived as unfair, but any such criticism would presumably apply to the efficiency gap as well.

If we do not advocate proportionality, why do we make this argument? Our claim is more subtle. By taking the mathematical principles of measuring "wasted votes," and calibrating this idea using a scenario in which there cannot be gerrymandering, we are led to an almost unmistakable conclusion, and one that differs significantly from the efficiency gap.[47] This merely suggests that the implications of the efficiency gap have not been carefully considered.

[45] Stephanolpoulos & McGhee, supra note 3, at 854.

[46] Brief of Eric McGhee as Amicus Curiae in Support of Neither Party, Gill v. Whitford (U.S. will argue Oct. 3, 2017) (No. 16-1161).

[47] In this sense, our argument differs significantly from other researchers who have studied the efficiency gap. *See* Cover, *supra* note 12, (referring to seats-votes proportionality as an important democratic norm), and Bernstein & Duchin, *supra* note 12, (criticizing the efficiency gap as penalizing proportionality). We do not assume that proportionality is a desirable feature of electoral outcomes or that a measure of partisan bias should be faulted for deviating from the proportionality norm. Our argument is that that

B. Scale Invariance and the Two-Seat Threshold

An efficiency gap of zero is essentially impossible due to randomness in the electoral process.[48] So that courts can apply the measure, its creators provide a test: redistricting legislation that results in an efficiency gap above a certain threshold should be presumptively illegal, subject to a second-stage judicial inquiry that they describe. They propose a specific threshold of two seats for congressional plans and 8% for state legislative plans,[49] and they claim that it would be "hard to deny" the reasonableness of this proposal even if "[s]cholars and judges may quibble" about the precise threshold.[50] We disagree. As we explain, the two-seat threshold is clearly an unreasonable measure of "how much partisan dominance is too much."[51]

The problem with the two-seat threshold is that it imposes a stronger constraint in large states than in small states on the number of seats that a party can be awarded. In fact, for the smallest states (those with no more than four representatives), the efficiency gap with the two-seat threshold imposes no constraint whatsoever on districting.[52] *Every* possible plan is acceptable.[53]

The 8% threshold that Stephanopoulos and McGhee propose for state legislative plans does not suffer from this scale-related problem. However, it and all other fixed thresholds suffers from a different problem. There is a possibility that it will reject every possible districting plan as being presumptively illegal.

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logic of the efficiency gap itself compels proportionality; consequently, it is hard to justify using the efficiency gap when a proportionality standard could be used in its place.

[48] Stephanopoulos & McGhee, *supra* note 3, at 887 (explaining that "almost every current plan" would not have an efficiency gap of zero and that "plans' efficiency gaps vary markedly from election to election").

[49] *Id.* at 884 (suggesting that "the bar be set at two seats for congressional plans and *8 percent* for state house plans"). The two-seat threshold is exactly 8% for states with twenty-five congressional districts, such as Florida following the 2000 Census.

[50] *Id.* at 897–98.

[51] *Id.* at 898, quoting League of United Latin Am. Citizens v. Perry, 548 U.S. 399, 420 (2006).

[52] To prove this statement, note that a districting plan exceeds the threshold if $(\mathbf{PS} - \frac{1}{2}) - \mathbf{2}\,(\mathbf{PV} - \frac{1}{2}) \geq \mathbf{T}$. This expression is true if and only if $\mathbf{PV} \leq \frac{1}{2}\,\mathbf{PS} + \frac{1}{4} - \frac{1}{2}\,\mathbf{T}$. Let $\mathbf{PS} = \mathbf{M}/\mathbf{N}$, where **M** is the number of seats won and **N** is the number of districts. Next, to win **M** districts requires that $\mathbf{PV} > \mathbf{M}/\mathbf{2N}$. Thus, the districting plan exceeds the threshold if $\mathbf{M}/\mathbf{2N} < \mathbf{PV} \leq \mathbf{M}/\mathbf{2N} + \frac{1}{4} - \frac{1}{2}\,\mathbf{T}$, which is possible if and only if $\mathbf{T} < \frac{1}{2}$. The two-seat threshold implies that $\mathbf{T} = 2/\mathbf{N} \geq \frac{1}{2}$ for $\mathbf{N} \leq 4$. ■

[53] The prior literature has noted that the efficiency gap does not work well in states with too few districts. Cover, *supra* note 12, notes that the a two-seat threshold cannot be used to invalidate districting plans in two-district states; Cho, *supra* note 12, argues that the efficiency gap can only take on few discrete values in states with few districts, a problem Bernstein & Duchin, *supra* note 12, refer to as "nongranularity." Our argument goes further and points out that this leads to a bias favoring a finding of partisan gerrymandering in large states as opposed to in small states.

The efficiency gap with a two-seat threshold is a test that declares a districting plan to be valid if a party has enough votes to justify the number of seats it has won. How many votes does a party need for a plan to pass?

We begin with a simple example of a five-district state. Suppose that the Republicans win 60% (or three) of the five districts. The Republicans cannot win 60% of the seats unless they have more than 30% of the statewide vote, as they need a majority in every district they win.[54] The efficiency gap formula tells us that they are below the threshold (and the plan passes the test) if they receive more than 35% of the vote.[55]

The difference between these numbers, in this case 5%, is what we call the "spare vote margin." It is the percentage of the vote share above the theoretical minimum (half the proportion of seats won) that a party needs for the plan to pass the test.

We have just established that, for the case of a five-district state and a party that wins three seats, the spare vote margin is 5%. What about the case of a five-district state and a party that wins all five seats? We know that the party cannot win 100% of the seats with fewer than 50% of the votes. The efficiency gap formula, meanwhile, tells us that the plan passes the test if the party's vote share exceeds 55%.[56] The difference between these numbers—the spare vote margin—is again 5%.

The fact that the spare vote margin was the same in these two cases is not a coincidence. In fact, as we prove, the spare vote margin is determined entirely by the number of districts in the state, and it is independent of the number of districts won by the party. For a state with **N** districts, the spare vote margin is expressed in terms of a percentage as:[57]

$$\mathbf{25 - \frac{100}{N}}.$$

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[54] They must win more than 50% of the votes in 60% of the districts; therefore, they must win more than 50% × 60% = 30% of the votes in the state.

[55] The districting plan exceeds the threshold if $(\mathbf{PS} - ½) - 2\,(\mathbf{PV} - ½) \geq \mathbf{T}$. In the example, $\mathbf{PS}$ = 60% = ⅗, and the threshold $\mathbf{T}$ = ⅖. Substituting these values, we get $(⅗ - ½) - 2\,(\mathbf{PV} - ½) \geq ⅖$, which is true if and only if $\mathbf{PV} \leq 0.35$ = 35%.

[56] The districting plan exceeds the threshold if $(\mathbf{PS} - ½) - 2\,(\mathbf{PV} - ½) \geq \mathbf{T}$. In the example, $\mathbf{PS}$ = 100% =1, and the threshold $\mathbf{T}$ = ⅖. Substituting these values, we get $(1 - ½) - 2\,(\mathbf{PV} - ½) \geq ⅖$, which is true if and only if $\mathbf{PV} \leq 0.55$ = 55%.

[57] For a threshold $\mathbf{T}$ and a vote share $\mathbf{PS}$, the spare vote margin is given by $\mathbf{PV}^* - ½\,\mathbf{PS}$, where $\mathbf{PV}^*$ satisfies $(\mathbf{PS} - ½) - 2\,(\mathbf{PV}^* - ½) = \mathbf{T}$. This latter expression reduces to $\mathbf{PV}^* = ½\,\mathbf{PS} + ¼ - ½\,\mathbf{T}$, which implies that the spare vote margin is $¼ - ½\,\mathbf{T}$. If we have $\mathbf{N}$ districts in the state, a two-seat threshold can be expressed as $2/\mathbf{N}$; by making the substitution $\mathbf{T} = 2/\mathbf{N}$ we get the expression $¼ - 1/\mathbf{N}$. Expressed in percentage terms, this is $25 - 100/\mathbf{N}$.

The spare vote margin gives us a relatively simple way to think about the consequences of changing the threshold. The larger the spare vote margin, the stronger the constraint imposed by the test. Because the spare vote margin is larger in larger states, this implies that the efficiency gap with a two-vote threshold constrains outcomes in large states more than it does in small ones. To see why, consider the case in which a party wins 80% of the seats with 50% of the vote. The plan will be approved as long as the party wins more than the spare vote margin plus 40% of the votes.[58] In a five-district state, 80% is four districts, and the spare vote margin is 5%; the plan is approved, as 50% is greater than 45%. In a ten-district state, 80% is eight districts, and the spare vote margin is 15%; thus the plan would fail, as 50% is less than 55%.

The efficiency gap provides no constraint at all unless the spare vote margin is strictly positive. This is because a party cannot win a district through a tie; it needs some spare votes to get a majority in the districts it wins. If the spare vote margin is zero (or negative), it is impossible for a party to win the seats yet have "too few" votes so as to fail the test. One can see from the formula that, with a two-seat threshold, the spare vote margin is negative if there are two or three districts in the state, and it is zero if there are four. It follows that this test is vacuous for small states: those with no more than four districts.

So the efficiency gap, with the two-seat threshold, treats large states differently from small states. In other words, it is "scale dependent." How strong is this dependency? Under the Congressional apportionment following the 2010 Census, twenty-nine states received five or more representatives. Figure 2 depicts the spare vote margins for these states. The smallest states (Connecticut, Oklahoma, and Oregon), with five representatives each, all have a spare vote margin of 5%, as we explained above. The next smallest states (Kentucky and Louisiana) have six representatives each, leading to a spare vote margin of 8⅓%. The largest states, California (with fifty-three representatives), Texas (thirty-six representatives), and New York and Florida (twenty-seven representatives each) have spare vote margins of over 23%, 22%, and 21%, respectively. The spare vote margin increases as the number of districts grows, but does so at a decreasing rate. Louisiana has 20% more representatives than Oklahoma, but its spare vote margin is 66⅔% larger; California has over 96% more representatives than New York, but its spare vote margin is larger by only 8½%.

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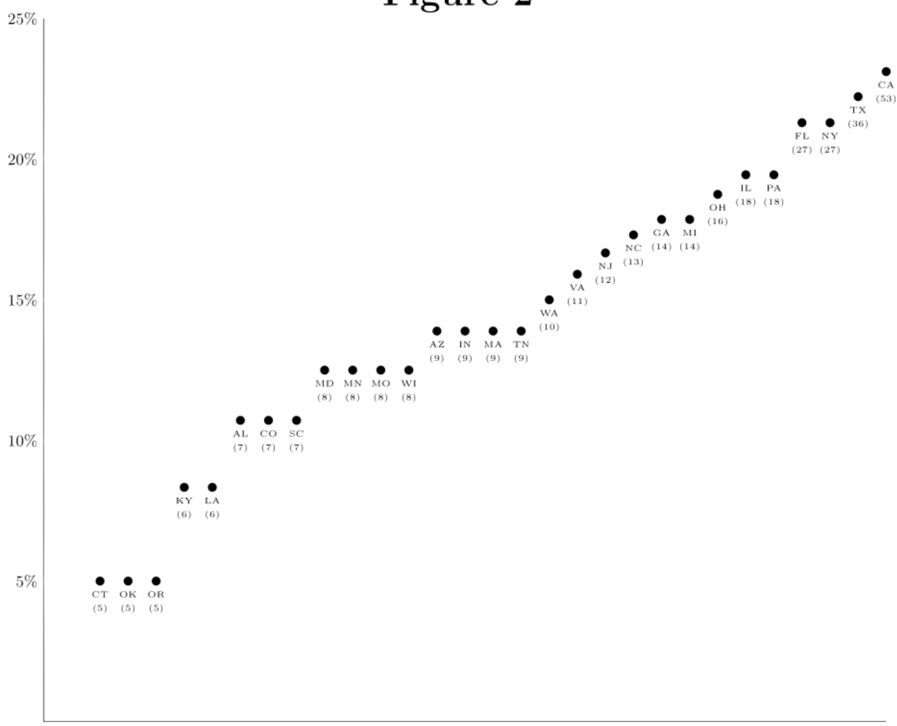
[58] As before, it is impossible for the party to win 80% of the seats unless it wins more than 40% of the votes. It must win 50% of the votes in 80% of the districts; 50% × 80% = 40%.

What does this mean in theory? Texas has six times the number of representatives as nearby Louisiana. Suppose that a party captures half of the seats with a minority of the popular vote. Will the districting plan be held to be presumptively invalid? The answer depends on whether that party's vote share exceeds a threshold equal to spare vote margin plus 25%. A party that wins half of the seats in Louisiana with 34% of the vote passes the test; a party that wins half of the seats in Texas with 47% of the vote fails the test. If a party wins two thirds of the seats (*i.e.* four out of six in Louisiana or twenty-four out of thirty-six in Texas), then it passes if it wins only 42% of the votes and is in Louisiana, but fails the test if it wins 55% of the votes and is in Texas.

Figure 2



A simple look at the data suggests that the likelihood that a state will exceed the two-seat threshold is correlated with the size of that state's population. From the data provided by Stephanopoulos and McGhee,[59] nine states

[59] Stephanopoulos & McGhee, *supra* note 3, at 879–81.

had plans that passed the two-seat threshold in the decade following the 2000 Census. These nine states included the eight most populous states plus Massachusetts. For the decade following the 1990 Census, there were six such states: five of the six largest states plus Washington, which curiously had a districting plan that exceeded the two-seat threshold in favor of the Republicans in one election and in favor of the Democrats in another. For the decade following the 1980 Census, there were again six such states: five of the seven largest in addition to Massachusetts. The pattern is less apparent in the decade that followed the 1970 Census: six of the ten states that passed the threshold were at the time among the nine most populous.

By contrast, for state legislatures, Stephanopoulos and McGhee suggest a threshold of 8%, independent of the size of the state or the number of legislative districts. An 8% threshold is equivalent to a two-seat threshold in the case where the state has exactly twenty-five districts. It is harder to see an obvious bias related to the size of the state; for the decade following the 2000 Census, for example, the 8% threshold was exceeded by three of the five most populous states (California, Florida, and New York, but not Pennsylvania or Texas), and by three of the five least populous states for which data is provided (Delaware, Vermont, and Wyoming, but not Alaska or Montana).[60]

We do not know whether a measure of gerrymandering should be entirely independent of the size of the state or the number of legislative districts. For example, if we were to agree that an imperfect measure was better than none, and that this measure is more sensitive to gerrymandering in small states than in large states, then it would make sense to adjust the threshold in larger states to account for this sensitivity. But it is hard to see how one can justify the extreme form of scale dependence that arises from the use of the two-seat threshold. The bias created is so strong that it would render congressional redistricting legislation in states such as Utah and Nevada immune from constitutional scrutiny.

We do not advocate an alternative to the two-seat threshold proposed by Stephanopoulos and McGhee. In part, this is because we do not believe that the efficiency gap produces a meaningful ranking of districts. But in part, to the extent that the efficiency gap may have some value, we do not believe that it can be combined with a threshold to create a meaningful test.

Any test of gerrymandering should ideally have two properties. First, it should sometimes reject districting plans; second, it should not reject all possible plans that could have been chosen. The first property requires that there

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[60] *Id.* at 882–84.

should be some combinations of electoral outcomes and districting plans that the test would reject as gerrymandered. We do not expect that a test be able to reject district plans regardless of the electoral outcome, but we think it is reasonable to expect that it sometimes be useful. The second property requires that there be no election for which every plan would be deemed to be presumptively illegal. The test of gerrymandering should not function as a "Catch-22."[61]

If the efficiency gap combined with a threshold is to satisfy the first property, the threshold must be strictly below 50%. This is because, by definition, the majority of votes in an election are not wasted, and therefore the efficiency gap must be strictly below 50%. A threshold above the maximum efficiency gap score would fail the first property because it could never reject a districting plan.

If the threshold is below 50%, however, we are led to a different problem. Recall the thought experiment where all voters vote Republican. All district plans in this state receive the same score, and that score is the worst possible, at nearly 50%. Every threshold below 50% will reject these plans, regardless of the outcome of the election.

An analogous problem arises in states that are not completely Republican. Consider, for example, a state where 80% of the votes for the state legislature are cast for Republicans. Such a state will be deemed to have an efficiency gap of at least 10% in favor of the Democrats. In this case, the 8% threshold advocated by Stephanopoulos and McGhee would lead to the rejection of all possible districting plans.[62]

We recognize that one might respond by saying that the efficiency gap is only intended to be used when the vote share is between 25% and 75%, and for states where there are at least five districts, although these claims do not appear in the Stephanopoulos and McGhee paper. There are several reasons why we would disagree with this defense of the measure. First, it ignores the nature of the thought experiment, which allows us to test our intuition in settings where the right answer is clear. We have no reason to believe that the failure of the efficiency gap in the thought experiment is not indicative of a broader problem that affects the measurement of partisan gerrymandering in large states where the vote share is between 25% and 75%.

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[61] JOSEPH HELLER, CATCH-22 (1961) (introducing the concept of the Catch-22).

Second, gerrymandering can still exist in states where one party has a large majority, especially if the state is large, and in states with four or fewer congressional districts. Democrats in Utah, for example, quite often believe that state Republicans attempt to gerrymander both the state legislative and federal congressional districts to minimize the Democrats' voting power.[63] Third, there is no clear justification for why 74% should be treated differently from 76%, or why a state with four districts is qualitatively different from a state with five. Last, but not least, none of these distinctions are necessary. As we have shown, the problems involving (a) states where one party is dominant and (b) small states is an artifact of choosing the wrong measure of benefit and the wrong threshold.

IV. THE PROBLEM OF PACKING AND CRACKING

In the previous part we asked whether the efficiency gap is a sensible measure of packing and cracking, under the assumption that packing and cracking is a good explanation of partisan gerrymandering. In this part, we challenge that assumption. We argue that the packing and cracking story ignores several considerations important to understand partisan gerrymandering, and we show that, as a result of these considerations, a naïve application of the efficiency gap can cause real harm.

A. Extremists

One of the most severe limitations of the efficiency gap is that it ignores many of the intricacies inherent to the political process, and instead summarizes a plan by two numbers: the number of seats awarded to each party, and the number of votes attained by each party. While these numbers reveal the number of Democrats and Republicans who get elected, they do not tell us what kind of Democrats and Republicans these are; that is, whether they are moderate, extreme, or somewhere in between. In other words, the efficiency gap does not contemplate that political parties may be heterogeneous.

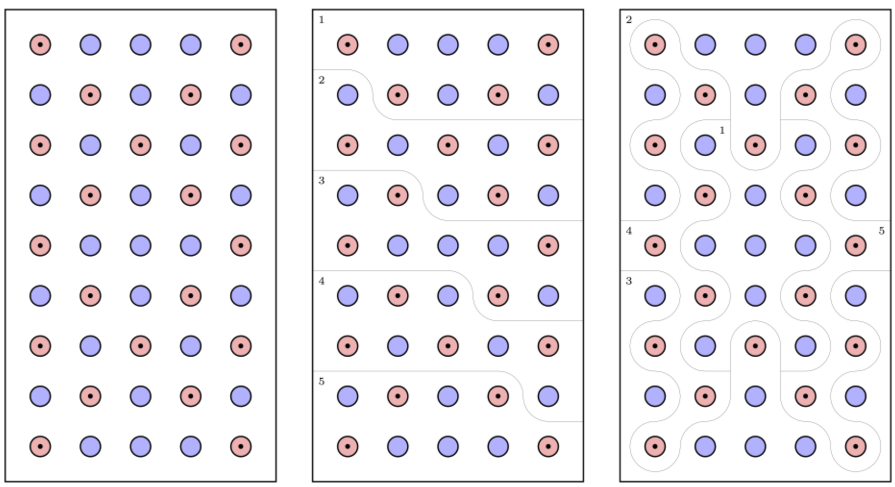
This is a problem because gerrymandering can affect not only which parties are elected, but also the specific political opinions of the representatives that comprise the legislature. The nature of these representatives is important; for example, the recent failure of the health bill was due to disagreements between Republican factions, each of whom believed that the Affordable Care Act was flawed.

[63] *See, e.g.*, Paul Rolly, *Rolly: Decades of Gerrymandering Bear Fruit: Utah's Legislature is Wacko*, SALT LAKE TRIB. (April 1, 2016), http://archive.sltrib.com/article.php?id=3727056&itype=CMSID.

Unfortunately, the use of the efficiency gap by courts can lead to results not anticipated by Stephanopoulos and McGhee. This is because the measure can favor plans that make it easier for political extremists to be elected, and which would naturally increase the level of political polarization in legislatures. More importantly, in spite of its proposed application in adjudicating Equal Protection cases, the use of the efficiency gap can actually harm the minority party.

To illustrate our point, we begin with a simple example of a five-district state, shown in Figure 3. The state has a slight majority of Democrats. In plan one (Figure 3(b)), each district has an identical distribution of voters; the Democrats win each of these districts with five of the nine votes. In plan two (Figure 3(c)), on the other hand, the state is divided into three safe-Democratic and two safe-Republican districts. This would lead to the legislature being composed of three Democrats and two Republicans.

Figure 3



(a) Before Districting (b) Plan One (c) Plan Two

Table 2 contains the efficiency gap calculations for these districting plans. The Republicans have over 44% of the vote, but receive no seats under plan one, so this plan is determined by the efficiency gap to be presumptively illegal, having been gerrymandered in favor of the Democrats, with a score of over 44%.[64] On the other hand, the Republicans receive two (or 40%) of the seats under plan two; as a consequence, this plan receives a perfect score of zero, and is presumptively legal. The efficiency gap thus provides a clear answer to the question of which districting plan is preferred.[65]

Table 2

		Number of Votes		**Wasted Votes**	
		Democrats	Republicans	Democrats	Republicans
Plan One	**District 1**	5	4	0	4
	District 2	5	4	0	4
	District 3	5	4	0	4
	District 4	5	4	0	4
	District 5	5	4	0	4
	Total	**25**	**20**	**0**	**20**
	efficiency gap: (20-0) ÷ 45 = 44.44% in favor of the Democrats				
Plan Two	**District 1**	9	0	4	0
	District 2	8	1	3	1
	District 3	8	1	3	1
	District 4	0	9	0	4
	District 5	0	9	0	4
	Total	**25**	**20**	**10**	**10**
	efficiency gap: (20-20) ÷ 45 = 0%				

[64] In fact, this is the worst possible districting plan according to the efficiency gap. Forty-four percent is above every threshold suggested by Stephanopoulos and McGhee. *See* Stephanopoulos & McGhee, *supra* note 3, at 885–91.

[65] Measures of partisan bias would generally give the opposite ranking, placing plan one ahead of plan two.

However, there are reasons to be suspicious of this result. First, while the Republicans win two seats under plan two, the Democrats still have a majority and can pass legislation over the objections of the Republicans. It is not clear why the seats make the Republicans better off. Second, the districts in plan one appear more competitive. One might think that politicians elected in politically lopsided districts may be more extreme than politicians from politically competitive districts, and hence the legislature resulting from plan two may be more polarized. The model used by Stephanopoulos and McGhee to motivate the efficiency gap describes voters and politicians only by their partisan affiliations, and does not provide a framework to distinguish between moderates and extremists.

To expand the analysis, we use the classical model of political competition introduced by the Scottish economist Duncan Black.[66] The model envisions political positions as being summarized by points on a line; each individual has a policy on the line that they prefer the most.[67] Voters care about policy positions in a straightforward matter, referred to as "single-peakedness" in the economics literature. As one moves further to the right from their most preferred policy, they are made less well-off; they are similarly harmed when the policy moves further to the left from their preferred point.

The model conforms to common informal descriptions, so that the "leftists" are further to the left on the line, while "rightists" are further to the right. Importantly, this model provides us with a language for describing some voters as having more extreme positions than others. This framework allows for much more generality than does a model that identifies people only by their affiliation as Republicans or Democrats. At the same time, by placing all of the voters on a line, it remains simple enough to provide us with powerful insights.

For example, Figure 4 depicts a line with four positions, the Leftists, the Center-Left, the Center-Right, and the Rightists. One can see that, relative to one another, these positions are placed along the line where we would

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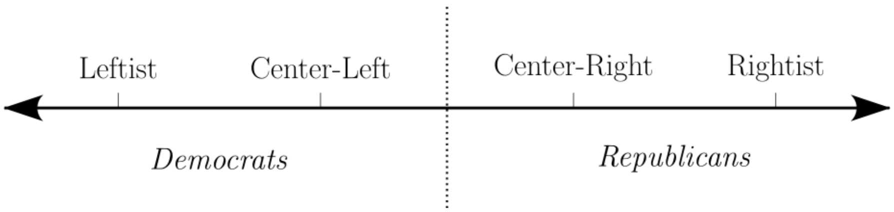
[66] DUNCAN BLACK, THE THEORY OF COMMITTEES AND ELECTIONS (1958). For an earlier application of this model in the context of economic competition, see Harold Hotelling, *Stability in Competition*, 39 ECON. J. 41 (1929).

[67] In Black's model, individual preferences are assumed to be what economists call "single peaked"; this means that they prefer politicians whose positions are closer to theirs on the line than those whose positions are further away. That is, given a choice between a Center-Left candidate or a Center-Right candidate, a Leftist will prefer the former to the latter. All Democrats will prefer a Center-Right candidate to a Rightist, if those are the choices. The notion of single-peakedness is slightly more general; it does not specify how an individual should rank two candidates, one on the individual's left and the other on the individual's right. BLACK, *supra* note 66, at 4–11.

naturally expect. The two more left-wing groups together comprise the Democratic Party, while the two more right-wing ones form the Republicans.

Figure 4



Using this model, Black proved the most basic result in formal political economy—the Median Voter Theorem—which states that under majority rule, the selected alternative will be the one chosen by the median voter. Recall that the median is a point for which half of the voters appear to the left, and half are to the right. In political competition within districts, the seat will be won by a politician whose position is located at the median of the preferred points of the voters on the line. The policy chosen by the legislature, in turn, will be the policy advocated by the median politician.

For example, let us start with a simple example: a five-seat legislature composed of three Democrats and two Republicans. Assume that two of the Democrats are Leftists and that one is from the Center-Left. The median legislator is the legislator from the Center-Left, because there are two legislators to her left (the two Leftists) and two to her right (the Republicans). The Median Voter Theorem predicts, then, that the chosen policy will be that of the Center-Left. This explains why the Republicans might prefer to have seats in the legislature. The Republican minority cannot get its own policy adopted, but its presence results in a more conservative policy than would be chosen by the Democrats alone.[68]

However, the intuition that more seats are better is not necessarily correct. Let us return to the example in Figure 3, but now, let us identify Democrats as either Leftist or Center-Left, and Republicans as either Center-Right or Rightist.[69] Two districting plans are shown in Table 3. These plans

[68] Recall that two out of three Democratic legislators are Leftists.

[69] In our example, the Leftist group is larger than the other three. Note that the term Leftist in this example is merely a label to denote the left-most 60% of the Democratic Party, while Center-Left denotes the more moderate 40%. Other labels could be chosen. Our example does not depend on the idea that the leftmost 60% forms a coherent political block; in practice, we think one would find a wide range of views

have the same efficiency gap scores as their counterparts in Figure 3; to see this, simply note that each district has the same total number of Democrats and Republicans, and consequently the efficiency gap calculation is described by the numbers in Table 2.

Table 3

	Democrats		**Republicans**	
Political Position	Leftist	Center-Left	Center-Right	Rightist
Number of Voters	15	10	10	10
	Plan One			
District 1	3	**2**	2	2
District 2	3	**2**	2	2
District 3	3	**<u>2</u>**	2	2
District 4	3	**2**	2	2
District 5	3	**2**	2	2
	Plan Two			
District 1	**5**	4	0	0
District 2	**5**	3	1	0
District 3	**<u>5</u>**	3	1	0
District 4	0	0	4	**5**
District 5	0	0	4	**5**

in both parties. We would be able to create similar examples in this case, where the elected representatives in plan one would be roughly at the median of all voters, while the elected representatives in plan two would be, roughly speaking, around the 30th and 80th percentiles, counting from the left.

As each district contains nine voters, the median voter in each district is the fifth voter, counting from either the left or the right. This is because there are four voters to the left of the median voter, and four voters to her right.

Voters in plan one are uniformly distributed; that is, each of the five districts has an equal number of members of each group. Because there are five Democratic voters in each district, the fifth voter must be among them; because there are only three Leftists in each district, that fifth voter cannot be a Leftist. It follows then that the fifth, or median, voter must be a member of the Center-Left. Because all five districts are identical, all five representatives are Democrats from the Center-Left.

Plan two has been engineered to create safe districts for the two parties. The three safe-Democratic districts (1–3) each contain five Leftists; this implies that the fifth voter is also a Leftist. The safe-Republican districts each contain five Rightists; this implies that the fifth voter in these districts must be a Rightist. Thus this plan will result in a legislature with three Leftist Democratic representatives and two Rightist Republican representatives.

The model predicts that the policies of the state legislature are determined by the median of the elected representatives. Under plan one, all representatives are members of the Center-Left, which implies that the plan would lead to Center-Left policies. Under plan two, the majority of legislatures are Leftists; this implies that the median, and consequently the chosen policy, is that of the Leftists.

Which outcome is better depends, of course, on one's political preferences. From the perspective of the Republicans, however, the consequences are clear. All Republicans prefer the first plan, even though every seat is captured by the Democrats.[70] This is because Republicans are better off with a Center-Left legislature than one controlled by the Leftists. The efficiency gap, however, favors the second district plan, as demonstrated in Table 2. This difference is important, because the efficiency gap has been touted as a way to stop partisan gerrymandering. Yet, under the pretense that it is fairer to Republicans, the efficiency gap would unambiguously choose the plan that makes Republicans worse off.

B. Uncertainty

Figures 1 and 3 depicted a story of gerrymandering in which politicians allocate voters into districts so as to maximize the number of seats their party

[70] The model ignores the possibility that some voters might like to hold office, and only considers the implemented policy.

receives. However, in practice, that story is not entirely accurate. The main problem is that politicians do not know where the voters are.

It is not that the politicians have no idea whatsoever about the voters' locations; otherwise, there would be little point in gerrymandering. They may have access to past election returns, the voters' partisan affiliations, census data including race and ethnicity, and other relevant information that they can use. However, this doesn't allow them to predict with certainty whom the voters will choose or whether they will show up at the polls.

The efficiency gap does not rely on the information used by the gerrymanderers, in part because it is not always clear what information they are using. Instead, it measures the efficiency of a districting plan using the actual votes from an election held under that plan. This is potentially problematic for many reasons.

First, it is not entirely clear which is the group that may be denied Equal Protection in an election. Is it the members of the Democratic party in the state? Or is it the group of people who chose to vote for the Democratic candidates in a particular election? If it is the former, then there is a problem in that it is hard to tell who are the party members from election data; it is known, after all, that voters do not always vote for their party's candidates. Furthermore, individuals vote differently across elections, and their voting decisions are themselves affected by the districting plan. If it is the latter, then there is a problem in that the relevant group changes from election to election, so evidence from 2014 may not be relevant in determining whether the districting plan denied Equal Protection to the group of Democratic voters in 2012. We are not the first to point out that this can be problematic; the volatility of the efficiency gap was an issue considered by the district court in *Whitford v. Gill*.

Second, the packing and cracking story upon which the efficiency gap was founded may not hold if the partisan redistricting committee does not know the outcomes of the elections in advance. Stephanopoulos and McGhee justify the efficiency gap by explaining that partisan gerrymanders are constructed through cracking and packing, and that, at "its core, [the efficiency gap] is nothing more than a tally of all the cracking and packing decisions in a district plan."[71]

However, it is by no means clear that packing and cracking is an optimal strategy in the presence of uncertainty. A sizable literature has arisen in economics and political science in which redistricting is treated as a classical optimization problem, where a partisan in control of redistricting wants to

[71] Stephanopoulos & McGhee, *supra* note 3, at 852.

maximize some numerical objective, usually the number of seats obtained by a party.[72] But these papers work in a framework of uncertainty, so that when districts are drawn, it is not known exactly how votes will be distributed. Statistical models and assumptions will govern these hypotheses, and the behavior of the partisan in the face of this uncertainty conforms to standard behavioral assumptions in the social sciences. The literature has not reached a consensus, largely because uncertainty can be understood in several different ways.[73] There is also research showing that the strategy is not optimal in the presence of geographic constraints.[74]

Third, judging a districting plan by the outcome of an election can itself be problematic. As is generally acknowledged, elections contain a significant random element. Using a single data point to determine whether the districting plan was a consequence of partisan gerrymandering is similar to trying to determine whether a coin is fair by flipping it once. For this reason, the Supreme Court in *Davis v. Bandemer* explained that one cannot establish partisan gerrymandering on the basis of a single election.[75] Stephanopoulos and McGhee do suggest that "sensitivity analysis" be done as part of the second stage of their test, but they do not explain how this should be done, leading us to suspect that the analysis may be based on rather artificial assumptions about voting behavior. It would make more sense to think about the randomness of the election in devising the test in the first place.

While the issue of randomness has been discussed, we believe that the problem is more serious than has been recognized.[76] In particular, we want to highlight two important problems. For the first, consider a state with two congressional districts, where 67% of the voters are Democrats. There are

[72] This literature was initiated by Owen & Grofman, *supra* note 22, who introduce the techniques of packing and cracking.

[73] *See, e.g.*, John N. Friedman & Richard T. Holden, *Optimal Gerrymandering: Sometimes Pack, but Never Crack*, 98 AM. ECON. REV. 113 (2008) (arguing that packing and cracking is not an optimal strategy under uncertainty); Faruk Gul & Wolfgang Pesendorfer, *Strategic Redistricting*, 100 AM. ECON. REV. 1616 (2010) (showing conditions under which a generalized form of the the pack-and-crack strategy is optimal); Hideo Konishi & Chen-Yu Pan, *Partisan and Bipartisan Gerrymandering* (B.C. Econ. Dep't, Working Paper No. 922, 2016) (hypothesizing that partisan redistricters may care about both the number of seats won and the political preferences of the politicians elected).

[74] Clemens Puppe & Atilla Tasnádi, *Optimal Redistricting Under Geographical Constraints: Why "Pack and Crack" Does not Work*, 105 ECON. LETTERS 93 (2009) (showing that the cracking and packing strategy may fail to be optimal when gerrymanderers face geographic constraints).

[75] Davis v. Bandemer, 478 U.S. 109, 135 (1986) ("Relying on a single election to prove unconstitutional discrimination is unsatisfactory."). The decision also hinted that two elections would still not be sufficient, by criticizing Justice Powell, who would, in their view, "allow a constitutional violation to be found where the only proven effect on a political party's electoral power was disproportionate results in one election (possibly two elections)." *Id.* at 141.

[76] A similar argument was raised by Cho, *supra* note 12, at 25 (noting that two different scenarios that can lead to identical efficiency gap scores).

two potential plans, shown in Table 4. Under the first plan, the Democrats win the first district by 51% and the second district by 83%; under the second plan, the Democrats win both districts by 67%.

Table 4

	Number of Votes	
	Democrats	Republicans
	Plan One	
District 1	51%	49%
District 2	83%	17%
	Plan Two	
District 1	67%	33%
District 2	67%	33%

From an ex ante perspective, these plans look very different. The first plan contains one "competitive" district, in which the Democrats received 51% of the vote.[77] One might view this as a sign that the legislature was willing to take more risk. Furthermore, the existence of a competitive district may have affected the electoral race and the resulting vote distribution. We do not claim that the first plan is better than the second; but they are clearly different. The efficiency gap, however, does not make a distinction between these plans; because the overall Democratic vote share is the same under both plans (67%) and because the Democrats win both seats, the two plans have identical efficiency gap scores.

[77] The district court in *Davis v. Bandemer* defined "competitive" districts as those in which "the anticipated split in the party vote is within the range of 45% to 55%." 478 U.S. at 130.

V. CONCLUSION

The efficiency gap is a flawed measure of partisan gerrymandering. Our analysis suggests that it has not been carefully thought through in the rush to have it enshrined in Constitutional law. The flaws are serious. The ranking produced by the efficiency gap does not make sense in even the simplest of settings. The application of the efficiency gap can increase polarization and weaken political competition. It can harm the weaker party not in charge of redistricting—despite its claim to help them.[78]

That we find flaws in the efficiency gap, however, does not mean that there are no judicially manageable standards that courts can use to test for partisan gerrymandering.[79] It means, only, that we must work harder to find them. Ideas should be carefully tested and subjected over time to serious critical examination before they are used to affect policy.

The challenge of stopping gerrymandering is not figuring out how to convince the Supreme Court. The challenge is in understanding gerrymandering so that we can clearly recognize it, and so that courts can recognize it, too. Until and unless meaningful standards are created to detect this behavior, or meaningful algorithms are developed to take redistricting out of the hands of partisans, it is difficult to blame the judges and the politicians for not eliminating the problem once and for all.[80]

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[78] We note that Cover, *supra* note 12, and Bernstein & Duchin, *supra* note 12, argue for a limited use of the efficiency gap by courts. They do not, however, provide much justification for their arguments. Cover argues that the efficiency gap does measure one significant democratic norm, without clearly explaining why the form of quasi-proportionality enshrined in the efficiency gap represents a significant democratic norm. Bernstein and Duchin argue that the efficiency gap can be useful when the critiques they identified do not apply, and that the court should endorse the efficiency gap "only as a first draft of a general standard that will be refined over time." Bernstein & Duchin, *supra* note 12, at 5. We are not as comfortable with this proposal. If adopted by the Supreme Court, the efficiency gap in its current formulation may significantly influence future redistricting to the voters' detriment. Furthermore, even if its application were limited to cases where their critiques (as well as ours and those of other researchers) do not apply, the lack of careful though put into this measure hints that there may be additional problems lurking beneath the surface. This and other proposals to limit gerrymandering through formulas require serious research before they should be applied by courts.

[79] We offer no opinion on the other standards employed by the district court in Whitford v. Gill, 218 F. Supp. 3d 837 (W.D. Wis. 2016).

[80] Of course, we can still blame the politicians who produce the heavily gerrymandered maps, just not those who refuse to ban the practice because they do not know how.